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2.6. Source/receiver flux calculations

The most general configuration of source/receiver geometry is presented in Fig. 2.12. For this situation the total flux received by the area A_o from the source area S_o can be determined using the formula [1]

$$\Phi = \int_{S_o} \int_{A_o} L \frac{ds_o \cos\theta da_o \cos\psi}{R^2} \tag{2.28}$$

where Φ is the total flux received by the area A_o from the source area S_o, θ is the angle made by the direction of emerging flux with respect to the surface of the source, ds_o is an infinitesimally small element of area at the point of definition in the source, dA_o is an infinitesimally small elements of area at the point of definition in the receiver, ψ is the angle made by the direction of coming flux with respect to the surface of the receiver, R is the distance between the emitting point of the source and the receiving point of the receiver.

Formula (2.28) is the fundamental equation describing the transfer of radiation from source to receiver. Many flux transfer problems involve this integration over finite areas of the source and the receiver. The problem can be quite complex analytically because in general L, θ, ψ, and R will be the functions of position in both the source and the receiver surfaces. There exists also general dependency of L on direction embodied in this equation, since the direction from a point in the source to a point in the receiver generally changes as the point in the receiver moves over the receiving surface.

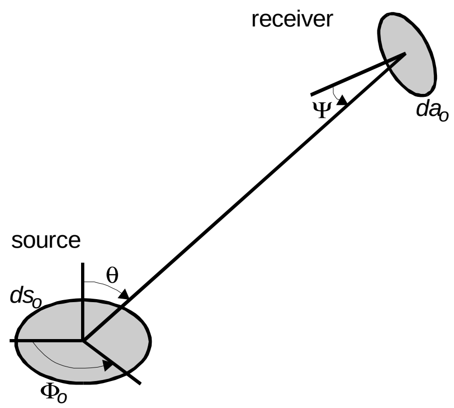

Fig. 2.12. General source/receiver geometry

We can distinguish two general cases of transfer of radiation from source to receiver useful for analysis of non-contact thermometers. First, when the radiation from the source comes directly to the detector. Second, when the radiation from the source comes to the detector through optical elements.

2. Geometry without optics

The equation (2.28) can be significantly simplified in three cases that are often met in practical radiation measurement.

For all three cases the angular dimension of detector is assumed to be small. This assumption is fulfilled in most practical cases. The differences between the mentioned above three cases are connected with size of source.

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For the first case we have a large circular or quasi-circular source irradiating a small area detector as shown in Fig. 2.13.

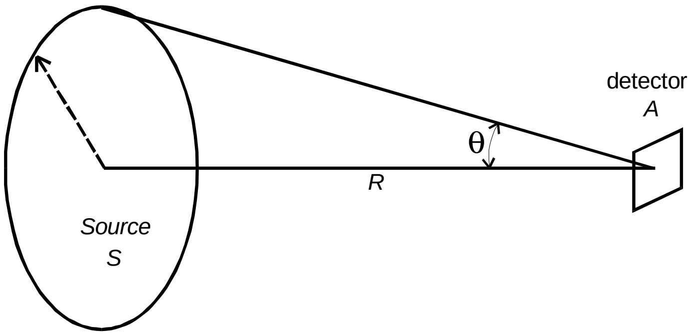

Fig. 2.13. Geometry of a circular source irradiating a small detector

For this case the flux received by the detector can be calculated from this formula

$$\Phi = \pi L \sin^2 \theta \times A \tag{2.29}$$

where Φ is the flux received by the detector, L is radiance of the source, θ is half of the angle that source subtends from the center of the detector, and A is the detector area.

For the second case a source of infinite size irradiates the detector. Then the angle θ=90°, sin θ=1 and the flux received by the detector can be calculated from this formula

$$\Phi = \pi L \times A. \tag{2.30}$$

For the third case, a small area source of dimensions much smaller than the distance R irradiates the detector. In this case the flux Φ can be calculated using another formula

$$\Phi = \frac{S \times L \times A}{R^2} \tag{2.31}$$

The presented above formulas (2.29 –2.31) are simple. Nevertheless they enable estimation of the flux obtained by a detector irradiated by many sources used in practice.

3. Geometry with optics

A block of optics before the detector is used in almost all non-contact thermometers. This block is used to limit the field of view of the detector and to increase the radiation that comes to the detector. Typical optical configuration during measurements with such thermometers is shown in Fig. 2.14.

Let us assume a typical situation when the detector is put exactly into the image plane of the tested object, and it can see, due to the cold shield, only the optics. Next, we assume that a well-corrected imaging aplanatic optics, that fulfils the sinus condition, is used.

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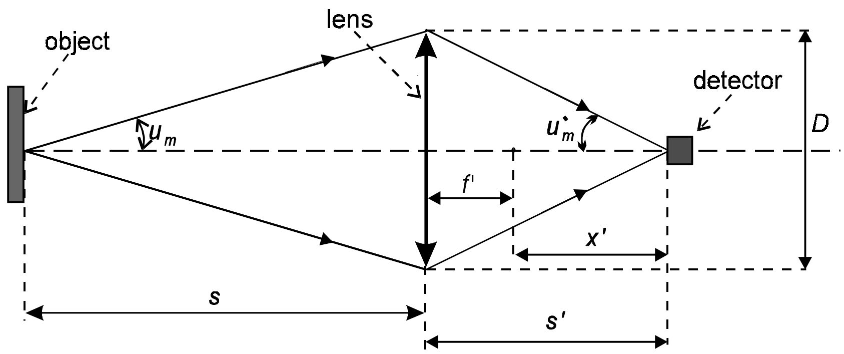

Fig. 2.14. Typical optical configuration during measurements with systems using an optical block before the detector

The radiation emitted by the surface determined by the detector angular dimensions with the solid angle determined by the area of the optical objective and distance optics-object reaches the detector. Derivation of formulas that enable calculation of the flux received by the detector, for the case shown in Fig. 2.14, on the basis of the fundamental equation (2.28) are complicated. Therefore, there will start the final formula that expresses the irradiation in the detector plane

$$E = \pi \tau_o L \sin^2 u_m' \tag{2.32}$$

where π is pi, τ_o is the optical transmittance, L is the object radiance, u_m' is the angle between the optical axis and the maximal aperture ray in the image space, $\sin u_m'$ is numerical aperture of the optical system in the image space.

Formula (2.32) is not too convenient as it requires knowledge about the parameter rarely known: the angle between the optical axis and the maximal aperture ray in image space u_m'. Therefore it is desirable to replace this angle with typical parameters describing optical system like its focal length *f'*, aperture diameter D and distance between the optics and the tested object *s*.
For situations presented in Fig. 2.14, using classical geometrical relationships and the well-known Newton formula we have

$$\sin u_m' = \frac{\frac{D}{2}}{\sqrt{\left(f' + \frac{f'^2}{s - f'}\right)^2 + \left(\frac{D}{2}\right)^2}} . \tag{2.33}$$

After putting Eq. (2.33) into Eq. (2.32) we obtain

$$E = \frac{\pi \tau_o L D^2}{4\left[\left(f' + \frac{f'^2}{s - f'}\right)^2 + \left(\frac{D}{2}\right)^2\right]} . \tag{2.34}$$

Formula (2.34) is equally general as formula (2.32) as it enables calculation of the radiant irradiance E for any distance s but it requires knowledge about only typical parameters of any imaging optics: D and *f'*.

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Now let us consider a case that quite often occur in many applications when the distance *s* is many times longer than the focal length *f'* (*s* >> *f'*). For such a situation the formula (2.34) simplifies to a new form

$$E = \frac{\pi \tau_o L}{4\left[F^2 + 1\right]}, \tag{2.35}$$

where *F* is the optics F-number that equal the ratio of the focal length *f'* and the aperture diameter *D.*

Many optical objectives used in non-contact thermometers, especially in thermal cameras, are systems of F-number higher or close to 2. For such systems F^2 >>1 and the formula enabling determination of the irradiance in the focal plane simplifies even further

$$E = \frac{\pi \tau_o L}{4F^2}. \tag{2.36}$$

Eq. (2.36) is used in derivation of many theoretical models of parameters of systems used to register optical radiation like *NETD*, *MRTD* or *MDTD* of infrared imaging systems. However, it is necessary to emphasize that Eq. (2.36) and the models derived from it are based on two important assumptions: the object is located in optical infinity (*s* >>> *f'*) and that optics of high *F*-number is used. When these two assumptions are not fulfilled the application of Eq. (2.36) can bring significant errors of estimation of the irradiance *E*. These assumptions are not fulfilled, for example, in optical microscopy where the distance *s* is short and *F* is low. Let us determine the irradiation *E* for such a case.

For a well designed optics that fulfils the sinus condition the lateral magnification of the optical system β equals

$$\beta = \frac{\sin u}{\sin u'}. \tag{2.37}$$

On the basis of geometry rules and the Newton formula we can derive a relationship between numerical aperture in imaging space $\sin u_m^{'}$ and the lateral magnification β

$$\sin^2 u_m^{'} = \frac{1}{4F^2(1+\beta)^2 + 1}, \tag{2.38}$$

that gives a new formula enabling determination of the irradiance *E*

$$E = \frac{\pi \tau_o L}{4F^2(1+\beta)^2 + 1}. \tag{2.39}$$

Eq. (2.39) shows that that the irradiance *E* decreases when the lateral magnification β of the optical system increases. Similarly, formula (2.34) suggests that the irradiance *E* decreases when the distance *s* decreases. Generally, both Eq. (2.34) and Eq. (2.39) show that the maximum irradiance *E* occurs when the measured object is in infinity and lateral magnification equals null (*s*=∝, β=0). For any other value of the distance *s* or lateral magnification β, the irradiance *E* will be lower. Let us define as magnification factor *MF* the ratio of the maximum irradiance *E* for theconditions *s*=∝, β=0 to the irradiance *E* for any other value of the *s* or β. The magnification factor *MF* can be determined using the following formulas

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$$MF = \frac{E(s=\infty)}{E(s)} = \frac{4\left[\left(f' + \frac{f'^2}{s-f'}\right)^2 + \left(\frac{D}{2}\right)^2\right]}{D^2\left(4F^2+1\right)}, \tag{2.40}$$

$$MF = \frac{E(s=\infty, \beta=0)}{E(\beta)} \frac{4F^2(1+\beta)^2+1}{4F^2+1}. \tag{2.41}$$

Graphical presentation of Eqs. (2.40-2.41) is shown in Fig. 2.15 - Fig. 2.16.

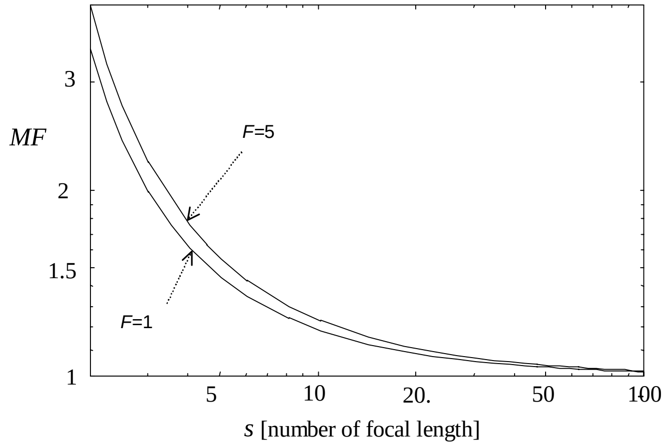

Fig. 2.15. Dependence of the magnification factor *MF* on the distance object-optics *s*

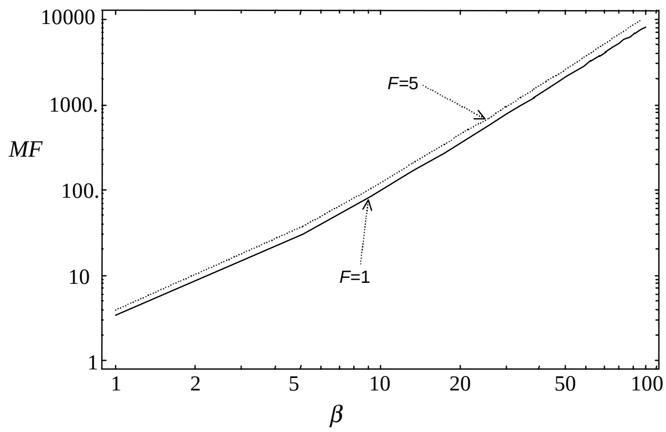

Fig. 2.16. Dependence of the magnification factor *MF* on the optical lateral magnification β

The magnification factor *MF* carries information how many times the irradiance *E* at optics focal plane is lower than for the ideal situation when the distance *s* equals infinity and the lateral magnification β equals null. From formulas (2.40) and (2.41) we can conclude that the factor *MF* varies from 1 to infinity and that it decreases with the distance *s* and increases with the lateral magnification β. As shown in Fig. 2.15 the dependence of *MF* on the distance *s* is significant for the distances *s* below about 20 *f'*. In case of infrared microscopy (Fig. 2.16), when the lateral magnification β>1, the values of *MF* are even higher. Consequences of these conclusions are significant as many projects to design thermal microscopes of high temperature resolution failed because of the presented dependence of the irradiance *E* on the magnification β.